Published by Institute of Physics Publishing for SISSA/ISAS

Received: *May 9, 2002*
Revised: *January 13, 2003*
Accepted: *February 14, 2003*

Yukawa hierarchy transfer based on superconformal dynamics and geometrical realization in string models

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Abstract: We propose a scenario that leads to hierarchical Yukawa couplings and degenerate sfermion masses at the same time, in the context of extra-dimensional models, which can be naturally embedded in a wide class of string models. The hierarchy of Yukawa couplings and degeneracy of sfermion masses can be realized thanks to superconformal gauge dynamics. The sfermion mass degeneracy is guaranteed by taking the superconformal fixed point to be family independent. In our scenario, the origin of Yukawa hierarchy is attributed to geometry of compactified dimensions and the consequent volume dependence of gauge couplings in the superconformal sectors. The difference in these gauge couplings is dynamically transferred to the hierarchy of the Yukawa couplings. Thus, our scenario combines a new dynamical approach and the conventional geometrical approach to the supersymmetric flavor problem.

Keywords: Beyond Standard Model, Supersymmetric Standard Model, Compactification and String Models.

It is a great challenge to derive the realistic fermion masses and their mixing angles from superstring models. Actually, one needs to explain hierarchical Yukawa couplings of quarks and leptons to electroweak Higgs fields,

$$y_{ij}\psi_i\psi_j H\,, \tag{1}$$

where ψ_i $(i = 1, 2, 3)$ represent three families of quarks and leptons collectively. Here we concentrate on models with softly-broken $\mathcal{N} = 1$ $D = 4$ supersymmetry (SUSY), and H denotes the up and down sectors of the Higgs fields, again collectively. Several mechanisms for generating hierarchical Yukawa couplings have been studied in the context of compactified string theories as well as four-dimensional effective field theories.

On the other hand, when one supposes that the SUSY survives to low-energy world, one should pay attention to the SUSY flavor problem; in particular if sfermion masses are not sufficiently degenerate, there appear unacceptably large flavor violations. One approach to avoid such large flavor violations is to assume that SUSY breaking sector is completely sequesterred from the Standard Model (SM) sector [1] and to find a flavor-blind mediation mechanism of SUSY breaking. An alternative approach is to suppose some nontrivial flavor dynamics that makes sfermions degenerate. It is this latter approach that we pursue here.

In this letter, we propose a new scenario that simultaneously realizes Yukawa hierarchy and sfermion mass degeneracy, independently of the origin of SUSY breaking. Our scenario is inspired by the work by Nelson and Strassler [2], in which strong dynamics of four-dimensional superconformal (SC) gauge theories plays an important role to achieve sfermion mass degeneracy as well as Yukawa hierarchy [3]–[6]. We combine this SC approach with a string-inspired mechanism based on geometry of extra dimensions.

Before explaining our scenario, let us first sketch generic features of purely string-theoretical mechanisms for generating hierarchical Yukawa couplings. In heterotic orbifold models, matter fields can be assigned to twisted closed string states, which are localized to different fixed points in the compactified space. Then their Yukawa couplings behave like $y_{ij} \sim e^{-af_{ij}^2}$, where f_{ij} is the distance between the fixed points corresponding to ψ_i and ψ_j [7]. See also ref. [8]. It does not seem easy, however, to obtain realistic Yukawa matrices only by this type of three-point couplings. On the other hand, in a model with intersecting D-branes [9], matter fields ψ_i and Higgs fields H arise from open strings between intersecting D-branes. Then their couplings behave as $y_{ij} \sim e^{-aA_{ijH}}$, where A_{ijH} is the area among three intersecting points corresponding to ψ_i, ψ_j and H. This approach may lead to realistic Yukawa matrices, but explicit analysis has not been done yet.

The string-theoretical approaches as above are interesting because they are purely geometrical in nature. Unfortunately, however, such approaches have a disadvantage from the viewpoint of the SUSY flavor problem. For instance, in the heterotic orbifold models, sfermion masses are not degenerate between different twisted sectors, except for the special case in which F-terms of the dilaton and overall moduli fields are the only source of SUSY breaking [10]. Generically, several moduli fields do contribute to SUSY breaking and sfermion masses become non-degenerate [11].

Alternatively, the origin of Yukawa hierarchy has been studied within the framework of effective field theories or string-inspired models. A well-known example is the Froggatt-

Nielsen (FN) mechanism [12], in which symmetry principle plays a role of controlling higher-dimensional couplings. In the most impressive version [13], an anomalous U(1) symmetry, which originates from string models [14, 15], is used to generate a suppression factor for the Yukawa couplings through the Fayet-Iliopoulos D-term. However, the problem in this approach, especially in models with anomalous U(1), is that sfermion masses suffer from flavor-dependent D-term contributions [16], which generically lead to large flavor violations.

Here, we take a recently-proposed approach based on field-theoretical dynamics, following the spirit of refs. [2]–[6]. At the cutoff scale Λ of the four-dimensional effective theory, we start from a non-hierarchical initial value of the Yukawa couplings, $y_{ij}(\Lambda) = \mathcal{O}(1)$. Within the SUSY framework, the renormalization group (RG) flow of y_{ij} can be written as

$$y_{ij}(\mu) = Z_{\psi_i}(\mu,\Lambda)\, Z_{\psi_j}(\mu,\Lambda)\, Z_H(\mu,\Lambda)\, y_{ij}(\Lambda)\,, \tag{2}$$

where $Z_\varphi(\mu,\Lambda)$ stands for the chiral wave-function renormalization factor of a superfield φ between Λ and low-energy scale μ. Note that our Z is the inverse of the usual one. Our goal is to have the desired pattern of Yukawa matrices by generating hierarchically small $Z_{\psi_i}(\mu,\Lambda)$ for the first and second families as a result of four-dimensional gauge dynamics.

Such a drastic RG flow cannot be realized in weakly-coupled theories like the minimal SUSY SM with the gauge couplings g_a ($a = 1,2,3$). In ref. [2], Nelson and Strassler have proposed coupling the SUSY SM sector (or its GUT-extensions) to the SC sector, which is a strongly-coupled gauge theory and is assumed to have an infrared fixed point [17, 18]. Here we concentrate on the SC sector with a product gauge group $G_{\rm SC} = \prod_i G_{\rm SC}^{(i)}$, whose gauge couplings we denote by g'_i ($i = 1,2,3$); we will omit the prime if no confusion is expected. Each family of quarks and leptons ψ_i couples to SC sector matter fields Φ_i and $\bar\Phi_i$, which are charged under the i-th SC gauge group $G_{\rm SC}^{(i)}$, through the 'messenger' coupling

$$\lambda_i\, \psi_i \Phi_i \bar\Phi_i\,. \tag{3}$$

Since the messenger interactions should be invariant under the SM-sector gauge group $G_{\rm SM}$, some SC-sector matter fields should also be charged under $G_{\rm SM}$. We will be a little bit more explicit on this point later.

Thanks to the messenger couplings (3), each quark and lepton ψ_i gains a large and positive anomalous dimension γ_{ψ_i} from the corresponding SC sector. Eventually, Yukawa couplings $y_{ij}(\mu)$ to electroweak Higgs fields are suppressed by the powers of anomalous dimensions. In the original Nelson-Strassler scenario, for instance, it is assumed that the first and second families couple differently to the SC sectors, so that different anomalous dimensions are generated for the first two families, $\gamma_{\psi_1} \neq \gamma_{\psi_2}$. Then one can realize hierarchical Yukawa couplings even though their initial values $y_{ij}(\Lambda)$ are similar at the cutoff scale. After generating the desired hieararchy in the Yukawa couplings, all the SC sectors are assumed to decouple at once at a certain intermediate scale M_C. Phenomenologically, the decoupling should be 'graceful' in the sense that there is no large threshold correction to the couplings and that no dangerous coupling nor bound state is generated.

The SC dynamics has another remarkable aspect. Within a pure superconformal field theory, soft SUSY breaking terms are exponentially suppressed towards the SC fixed point, as was first shown (for SQCD and its dual) in refs. [19, 20]. For an SC model coupled with the SUSY SM sector, where the SM gaugino masses M_a $(a = 1, 2, 3)$ are not suppressed, the mass-squared matrix of each sfermion $\widetilde{\psi}_i$ converges, for any initial values, on [3, 4]

$$m^2_{\tilde{\psi}_i\tilde{\psi}_j}(M_C) \;\longrightarrow\; \frac{\delta_{ij}}{\Gamma_{\psi_i}} \sum_a 4C\Big(R^{(a)}_\psi\Big)\, \alpha_a(M_C)\, M^2_a(M_C)\,, \tag{4}$$

which is one-loop suppressed and flavor diagonal before diagonalizing Yukawa matrices. Here $\alpha_a \equiv g_a^2/8\pi^2$ and $C(R^{(a)}_\psi)$ is the quadratic Casimir coefficient. The factor Γ_{ψ_i} is determined by anomalous dimension γ_{ψ_i}; once we know γ_{ψ_i} as a function of g'_i and λ_i, we can proceed the grassmanian expansion of γ_{ψ_i} in terms of background superfields, which are SUSY extensions of couplings g'_i and λ_i [21]. Thus, the convergence values depend on $\Gamma_{\psi_i} \sim \gamma_{\psi_i}$. When the anomalous dimensions are different between the first and second families, there remains slight non-degeneracy of sfermion masses,

$$m^2_{\tilde{\psi}_1}(M_C) - m^2_{\tilde{\psi}_2}(M_C) = \left(\frac{1}{\Gamma_{\psi_1}} - \frac{1}{\Gamma_{\psi_2}}\right) \sum_a 4C\Big(R^{(a)}_\psi\Big)\, \alpha_a(M_C)\, M^2_a(M_C)\,. \tag{5}$$

Note that this non-degeneracy is one-loop suppressed. Thus, if radiative corrections due to the SM gaugino masses, which are of course flavor-blind, are large enough, these sfermion masses are sufficiently degenerate at the weak scale. However, such radiative correction is small for slepton masses, especially for right-handed sleptons, as was estimated in refs. [3, 4]. See also ref. [6] for subtleties of the evaluation (4) and GUT case.

It is the infrared convergence property of sfermion masses that makes the SC approach attractive compared with the conventional approaches based on geometry or symmetry. In addition to the original Nelson-Strassler scenario, there exist several ways of modification as we shall show later. Moreover, a modified version of the scenario can be realized in string models in a natural manner.

We first show how the present SC framework, the SUSY SM coupled with product-type SC sectors, can be realized in string models. To this end, we take a type IIB orientifold (type I) model with $D9$-branes and $D5$-branes, where the extra six-dimensional space is compactified on $T^2_1 \times T^2_2 \times T^2_3$ and further orbifolded by a discrete symmetry [23, 24]. We assume that the SM gauge groups $G_{\rm SM}$ originate from the $D9$-branes. Other open string states that have both ends on the $D9$-branes are classified into three sectors, C^{99}_m $(m = 1, 2, 3)$, which have a nonzero momentum along the m-th torus T^2_m. Now let us assign the i-th family of quarks and leptons ψ_i to the C^{99}_i sector, i.e., we identify $i = m$. In addition, we need the SC sectors. Here we assume that the SC gauge group $G_{\rm SC} = \prod_i G^{(i)}_{\rm SC}$ originates from $D5$-branes. There are three types of $D5$-branes, $D5_i$, wrapping around T^2_i, and we assign $G^{(i)}_{\rm SC}$ to the gauge theory on $D5_i$. Furthermore, there is an open string sector connecting the $D9$ and $D5_i$ branes. Only in this sector, denoted by C^{95_i}, matter fields are charged under both $G_{\rm SM}$ and $G^{(i)}_{\rm SC}$. Therefore a natural candidate for the SC matter fields, Φ_i and $\bar{\Phi}_i$, is provided by the C^{95_i} sector. With this choice, the messenger couplings of the form (3) are automatically realized[1] since it is known [23, 25] that among the general

[1] In fact, we do not need such a tight selection rule. A discussion on the Yukawa hierarchy transfer scenario in the presence of mixed messenger couplings, $\lambda_{ij}\psi_i\Phi_j\bar{\Phi}_j$, will be presented in a separate publication [22].

trilinear couplings $\lambda^i_{jk} C_i^{99} C^{95_j} C^{95_k}$, only the followings are allowed by the stringy selection rule,

$$\lambda^i_{jk} = g_9\, \delta^i_j \delta^i_k\,, \tag{6}$$

where g_9 is the four-dimensional gauge coupling on the $D9$-branes. This completes our assignment of gauge groups and matter fields. Observe that three complex dimensions just fit in the existence of three families in Nature, and more specifically, the product-group structure of the SC sector naturally arises in this setup. A similar setup can be realized in a model with $D3$-branes and $D7_i$-branes.

Our string-theoretical realization of the SC framework suggests a peculiar initial condition of the couplings at the cutoff Λ, which we identify with the string scale. In ten dimensions, the gauge coupling is determined by a vacuum expectation value (VEV) of the dilaton field, but our gauge groups $G_{\rm SM}$ and $G_{\rm SC}^{(i)}$ originate from different branes, which can have different volume in the extra-dimensional space. Actually, the four-dimensional gauge couplings on the $D9$-brane and the $D5_i$-brane, g_9 and g_{5_i}, are related by

$$\frac{1}{g_{5_i}^2} = \left(\frac{V_i}{V}\right)\frac{1}{g_9^2}\,, \qquad \text{i.e.,} \qquad g_9^2 = \frac{g_{5_1}^2}{V_2V_3} = \frac{g_{5_2}^2}{V_3V_1} = \frac{g_{5_3}^2}{V_1V_2}\,, \tag{7}$$

where V_i is the volume of T_i^2 (in the Planck unit) and $V = V_1V_2V_3$. For a given value of the $D9$-brane coupling g_9, the value of each gauge coupling g_{5_i} depends on the volume factor of the codimension-four space transverse to the $D5_i$-branes. Therefore, *the SC gauge couplings can be different with each other, depending on the geometry of compactified dimensions*. Moreover, if V_3 is large, the first and second SC sectors naturally become strongly coupled. Note that this relation (7) holds for holomorphic gauge couplings in the effective field theory, since these couplings are determined by (the real part of) VEVs of chiral superfields, the dilaton and moduli fields.

Next we study a possible origin of Yukawa hierarchy. For concreteness we take the $G_{\rm SC}^{(i)}$ gauge theory to be an SU$(N_{\rm c}^{(i)})$ with $N_{\rm f}^{(i)}$ number of flavors; Φ_i and $\bar\Phi_i$ belong to the fundamental and anti-fundamental representations under SU$(N_{\rm c}^{(i)})$. We also assume that Φ_i and $\bar\Phi_i$ are the only charged matter fields in each SC sector. The $G_{\rm SC}^{(i)}$ gauge theory has a superconformal fixed point if $(3/2)N_{\rm c}^{(i)} < N_{\rm f}^{(i)} < 3N_{\rm c}^{(i)}$, where $N_{\rm f}^{(i)}$ includes the dimensions of Φ_i and $\bar\Phi_i$ under $G_{\rm SM}$. For the initial condition of the couplings, we will be more general for a moment than what is suggested by the above stringy realization, to list various possibilities for generating Yukawa hierarchy.

To find the suppression factor $Z_{\psi_i}(M_C,\Lambda)$ of the Yukawa couplings y_{ij} at the decoupling scale M_C, let us write the RG flow of the messenger coupling $\lambda_i(\mu)$ above the scale M_C as

$$\lambda_i(\mu) = Z_{\psi_i}(\mu,\Lambda)\, Z_{\Phi_i\bar\Phi_i}(\mu,\Lambda)\, \lambda_i(\Lambda)\,. \tag{8}$$

The factor $Z_{\Phi_i\bar\Phi_i} \equiv Z_{\Phi_i}Z_{\bar\Phi_i}$ can be evaluated by integrating the RG equation for the SC gauge coupling α_i. Equivalently, we use the relation between the holomorphic and physical gauge couplings, $\widehat\alpha_i$ and α_i, in the SC sector [26, 27],

$$\frac{1}{\widehat\alpha_i} + N_{\rm f}^{(i)} \ln Z_{\Phi_i\bar\Phi_i} = F(\alpha_i) \equiv \frac{1}{\alpha_i} + N_{\rm c}^{(i)} \ln\alpha_i + \cdots\,. \tag{9}$$

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The function F may depend on Yukawa couplings, but we neglect such dependence here. Using the exact result $\widehat{\alpha}^{-1}(\mu) - \widehat{\alpha}^{-1}(\Lambda) = (3N_{\rm c} - N_{\rm f})\ln(\mu/\Lambda)$ for the holomorphic coupling, the factor $Z_{\Phi_i\bar{\Phi}_i}(\mu,\Lambda)$ is evaluated to be

$$Z_{\Phi_i\bar{\Phi}_i}(\mu,\Lambda) = \frac{Z_{\Phi_i\bar{\Phi}_i}(\mu)}{Z_{\Phi_i\bar{\Phi}_i}(\Lambda)} = \left(\frac{\mu}{\Lambda}\right)^{-\gamma_*^{(i)}} \exp\left[-\frac{F(\alpha(\Lambda)) - F(\alpha_i(\mu))}{N_{\rm f}^{(i)}}\right], \tag{10}$$

where $\gamma_*^{(i)} \equiv (3N_{\rm c}^{(i)} - N_{\rm f}^{(i)})/N_{\rm f}^{(i)}$ is the absolute value of the anomalous dimension of SC matter fields at the fixed point. Substituting this expression (10) into eq. (8), we find that the suppression factors of the Yukawa couplings (2) are given by a formula

$$Z_{\psi_i}(\mu,\Lambda) = \left(\frac{\mu}{\Lambda}\right)^{\gamma_*^{(i)}} \times \left[\frac{\lambda_i(\mu)}{\lambda_i(\Lambda)}\right] \times \exp\left[\frac{F(\alpha_i(\Lambda)) - F(\alpha_i(\mu))}{N_{\rm f}^{(i)}}\right]. \tag{11}$$

In this formula, we can replace $\lambda_i(\mu)$ and $\alpha_i(\mu)$ with their fixed point values, $\lambda_*^{(i)}$ and $\alpha_*^{(i)}$, unless the initial couplings are so small that the fixed point is not reached for $\mu \geq M_C$.

The equation (11), together with eq. (2), is a master formula for Yukawa hierarchy in the present SC approach, i.e., by coupling the SM sector to the SC sector with product gauge group. Corresponding to three factors on the right-hand side of this formula, there are three possibilities to achieve the hierarchical Yukawa matrices (provided that all the SC sectors decouple at the same scale M_C). The first possibility is the Nelson-Strassler scenario, in which all the couplings at the cutoff scale Λ have no hierarchy, but the SC-sector gauge theories have family-dependent fixed points and anomalous dimensions. Thus, the Yukawa couplings $y_{ij}(M_C)$ are suppressed in a flavor-dependent manner as

$$Z_{\psi_i}(M_C,\Lambda) \sim \left(\frac{M_C}{\Lambda}\right)^{\gamma_*^{(i)}}. \tag{12}$$

In this case, the origin of Yukawa hierarchy is purely dynamical. As we mentioned before, however, we have non-degeneracy (5) of sfermion masses although it is one-loop suppressed.

The second possibility, a scenario of *Yukawa hierarchy transfer*, was proposed in ref. [5]. In order to realize sufficient degeneracy of sfermion masses, the same structure of SC sectors was assumed there; the same gauge group, the same field content and thus the same fixed point. The origin of Yukawa hierarchy was attributed to hierarchical initial values of the messenger couplings, $\lambda_1(\Lambda) > \lambda_2(\Lambda) \gg \lambda_3(\Lambda)$, which are (inversely) transferred by family-independent SC dynamics to the desired hierarchy of $y_{ij}(M_C)$, according to

$$Z_{\psi_i}(M_C,\Lambda) \sim \left(\frac{M_C}{\Lambda}\right)^{\gamma_*} \frac{\lambda_*}{\lambda_i(\Lambda)}. \tag{13}$$

Furthermore, the assumed initial hierarchy can be realized, without spoiling sfermion mass degeneracy, by using the FN mechanism in SC sector. [As was explained in ref. [5], there is no room for the FN mechanism in the SM sector.] Thus, this scenario is a hybrid of the SC approach and the conventional mechanism based on symmetry principle.

Now, we point out the third possibility for the Yukawa hierarchy, which can be most naturally realized in the string-theoretical setup described before. The idea is to combine

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a dynamical mechanism based on SC gauge theories with a mechanism based on geometry in extra dimensions. Suppose that as in the second scenario, each SC sector has the same gauge group and the family-independent fixed point, $\alpha_*^{(i)} = \alpha_*$ and $\lambda_*^{(i)} = \lambda_*$. In addition, we apply the stringy initial condition; this time, the initial condition (6) of the messenger couplings is not hierarchical, $\lambda_i(\Lambda) = g_9$, but the initial values of gauge couplings $\widehat{\alpha}_i(\Lambda)$ can be different. Indeed, in our string-theoretical realization, we have, by using the relation (9) with $Z_{\Phi\bar{\Phi}}(\Lambda) = 1$ as well as the relation (7),

$$F(\alpha_i(\Lambda)) = \frac{1}{\widehat{\alpha}_i(\Lambda)} = \frac{8\pi^2}{g_{5_i}^2} = \left(\frac{V_i}{V}\right)\frac{8\pi^2}{g_9^2}\,, \tag{14}$$

where in the second equality, we have made the tree-level matching of gauge couplings between the effective and string theories. Substituting this into the master formula (11), we finally arrive at

$$\varepsilon_i \equiv Z_{\psi_i}(M_C,\Lambda) = C\exp\left[\frac{8\pi^2}{N_{\rm f}\,g_{5_i}^2}\right] = C\exp\left[\frac{8\pi^2}{N_{\rm f}\,g_9^2}\left(\frac{V_i}{V}\right)\right], \tag{15}$$

where C is a universal suppression factor

$$C \equiv \left(\frac{M_C}{\Lambda}\right)^{\gamma_*}\left(\frac{\lambda_*}{g_9}\right)\exp\left[-\frac{F(\alpha_*)}{N_{\rm f}}\right]. \tag{16}$$

Thus, the wave-function suppression factor $\varepsilon_i = Z_{\psi_i}(M_C,\Lambda)$ for the i-th family of quarks and leptons depends on geometric data of the compactification, i.e., the volume V_i of the i-th torus. As the volume V/V_i of the transverse space of the i-th $D5$-brane becomes large, the initial value of the $G_{\rm SC}^{(i)}$ gauge coupling becomes large and the factor ε_i becomes less suppressed.

In this way, the (holomorphic) gauge couplings in SC sectors can be different at the string scale Λ if the compactified extra-dimensional space is anisotropic, and such difference is again transferred by flavor-independent superconformal dynamics to the desired hierarchy of Yukawa couplings of quarks and leptons. For example, the Cabbibo angle can be explained if the size of the first and second tori is slightly different in such a way that

$$\varepsilon' \equiv \frac{\varepsilon_1}{\varepsilon_2} = \exp\left[\frac{8\pi^2}{N_{\rm f}\,g_9^2}\left(\frac{V_1 - V_2}{V}\right)\right] \sim 0.22\,. \tag{17}$$

Moreover, if only the volume V_3 of the third torus is large (in the Planck unit),

$$V_3 \gg V_2 > V_1 \sim 1\,, \tag{18}$$

the corresponding gauge coupling $g_3' \sim g_{5_3}$ is as small as the SM-sector gauge couplings $g_a \sim g_9$ and will not reach the fixed point. Then the C_3^{99} sector of quarks and leptons do not receive much suppression, leaving the top Yukawa coupling of order one.

As an illustration, we give a toy model with $G_{\rm SM} = {\rm SU}(3)_C$ and three families of quarks. Now the SC matter fields Φ_i and $\bar{\Phi}_i$ belong to $(\mathbf{3}+\overline{\mathbf{3}}, \boldsymbol{N}_{\mathbf{c}})$ and $(\mathbf{3}+\overline{\mathbf{3}}, \overline{\boldsymbol{N}}_{\mathbf{c}})$ under $G_{\rm SM}\times G_{\rm SC}^{(i)}$ so that the messenger interactions (3) are trivially gauge invariant. We assign

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the up sector of quarks $u_{\mathrm{L,R}i}$ to the C_i^{99} sector, respectively for $i = 1, 2, 3$. If we assume that $V_1 < V_2 \ll V_3$, then the up-sector Yukawa matix at the scale M_C takes the form

$$y_{uij} \sim \begin{pmatrix} \varepsilon_1^2 & \varepsilon_1\varepsilon_2 & \varepsilon_1 \\ \varepsilon_1\varepsilon_2 & \varepsilon_2^2 & \varepsilon_2 \\ \varepsilon_1 & \varepsilon_2 & 1 \end{pmatrix} . \tag{19}$$

As for the down sector of quarks, we assign the left-handed quarks $d_{\mathrm{L}i}$ as above. A realistic Yukawa matrix can be obtained by assigning their right-handed quarks $d_{\mathrm{R}1}$, $d_{\mathrm{R}2}$ and $d_{\mathrm{R}3}$ to C_1^{99}, C_2^{99} and C_2^{99}, respectively,

$$y_{dij} \sim \varepsilon_2 \begin{pmatrix} \varepsilon'\varepsilon_1 & \varepsilon_1 & \varepsilon_1 \\ \varepsilon'\varepsilon_2 & \varepsilon_2 & \varepsilon_2 \\ \varepsilon' & 1 & 1 \end{pmatrix} . \tag{20}$$

Here we have taken the factor C common, but in principle these can be independent for left and right, and/or up and down sectors. In this way, realistic Yukawa matrices can be obtained even in this simple toy model (as far as quark sector is concerned).

We have not taken into account the stringy selection rule among $C_i^{99}C_j^{99}C_k^{99}$ couplings. Unfortunately, the stringy selection rule allows only the coupling with $(i, j, k) = (1, 2, 3)$ and its permutations. Therefore, if the electroweak Higgs fields are assigned to one of C_i^{99} sectors, some of necessary Yukawa couplings are not allowed. A possible wayout is to assume that the light Higgs fields would be linear combinations of Higgs fields from several sectors. Alternatively, one could suppose that for some reasons, effective field theory does not respect all of the stringy selection rules. As a matter of fact, in most of string models like those sketched at the beginning, the selection rules are too restrictive for three-point couplings as well as higher-point couplings [28]. It remains a challenging problem to derive realistic Yukawa couplings from string theory, especially with the minimal number of Higgs fields.

So far we have assumed that all the SM fields originate from C^{99} sectors. A certain explicit string construction, however, suggests that some of the SM matter fields, e.g. the right-handed quarks, come from C^{95_i} sectors [29]. Suppose that some of SM fields come from C^{95_i}, e.g. $i = 3$. Then in the present simple setup, these fields do not couple to the SC sectors. Thus their Yukawa couplings do not change drastically from their initial values and are accordingly expected to remain $O(1)$. It seems to be a problem in the realistic model building. In this setup, we have to take more complicated SC sectors, or extend the SM gauge group. We do not discuss this case for this is beyond the scope of the paper.

Finally we compare our string-inspired scenario with the purely geometrical approaches. The main difference lies in the sfermion mass spectrum. At the cutoff scale Λ, we have generically non-universal sfermion masses among three C_i^{99} sectors to which three families of quarks and leptons belong. For example, when the SUSY is broken by F-components of dilaton and moduli fields, soft scalar masses can be parametrized as [25]

$$m^2_{C_i^{99}} = m^2_{3/2}\left(1 - 3\Theta_i^2 \cos^2\theta\right) , \tag{21}$$

where $m_{3/2}$ is the gravitino mass and θ and Θ_i are goldstino angles. In the conventional approaches, sfermion masses at the weak scale are also non-degenerate for generic parameter space. In our scenario of string-inspired hierarchy transfer, on the other hand, the family-independent structure of SC dynamics guarantees the denegeracy below the decoupling scale M_C of the superconformal field theories.

To summarize, we have proposed an extra-dimensional scenario that simultaneously leads to hierachical Yukawa couplings and degenerate sfermion masses. Our scenario combines the conventional geometrical approach to Yukawa hierarchy with a new dynamical approach based on superconformal field theories, and can have a natural realization in string models.[2] The volume suppression in extra-dimensional spaces can generate the difference among initial values of SC gauge couplings, and such difference is transferred by the superconformal dynamics to the hierarchy of Yukawa couplings of quarks and leptons. The same dynamics makes sfermion masses degenerate.

In this letter, we have confined ourselves to the SC sector of product-type and assumed their simultaneous decoupling from the SM sector. A concrete mechanism for the decoupling as well as the 'unification' of SC sectors into a simple group will be discussed elsewhere.

Acknowledgments

The authors would like to thank Yoshihisa Yamada for discussions. The work of T.K. is supported in part by Grant-in-Aid for Scientific Research from Ministry of Education, Science, Sports and Culture of Japan (#14540256). The work of T.N. is supported in part by the Japan Society for the Promotion of Science under the Predoctoral Research Program.

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